Abstract

Let f be a positive definite ternary quadratic form of level N. Let $r(f,n)$ be the number of representations of n by f and $r(\mathrm{spn} f, n)$ the weighted mean over all representation numbers of forms in the spinor genus of f. If we write $n = 2^{e_2} \prod_{p \geq 3} p^{e_p}$, it is shown

$$r(f,n) = r(\mathrm{spn} f, n) + O_{e_2,\varepsilon}(N^{45/28} n^{1/2-1/28+\varepsilon})$$

for any $\varepsilon > 0$, as well as the uniform bound

$$r(f,n) = r(\mathrm{spn} f, n) + O_{\varepsilon}(N^{A} n^{1/2-1/28+\varepsilon})$$

with an absolute effective constant A. This extends a result of Duke and Schulze-Pillot in making explicit the dependence on the form f in the error term. As an application the number of representations of n as a sum of three squareful integers is considered.

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