ON THE CENTRAL VALUE OF SYMMETRIC SQUARE L-FUNCTIONS

VALENTIN BLOMER

ABSTRACT. Let $S_k(N,\chi)$ be the space of cusp forms of weight k, level N and character χ. For $f \in S_k(N,\chi)$ let $L(s,\mathrm{sym}^2 f)$ be the symmetric square L-function and $L(s, f\otimes f)$ be the Rankin-Selberg square attached to f. For fixed $k \geq 2$, N prime, and real primitive χ, asymptotic formulas for the first and second moment of the central value of $L(s,\mathrm{sym}^2 f)$ and $L(s, f\otimes f)$ over a basis of $S_k(N,\chi)$ are given as $N \to \infty$. As an application it is shown that a positive proportion of the central values $L(1/2,\mathrm{sym}^2 f)$ does not vanish.

2000 *Mathematics Subject Classification.* 11F66, 11F67, 11M41.

Key words and phrases. symmetric square L-function, nonvanishing, mollification, central value, Rankin-Selberg L-function.